

Introduction aux Théories de la Gravitation

Cours: Physique Master 1 (ENS)



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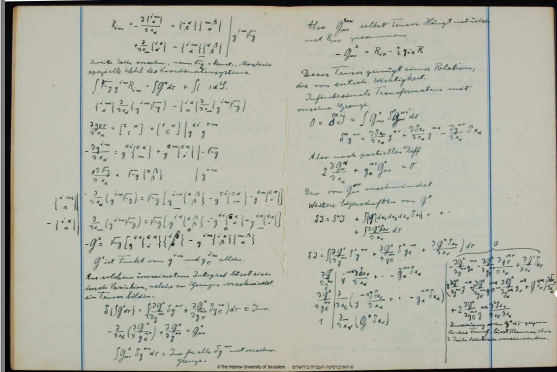
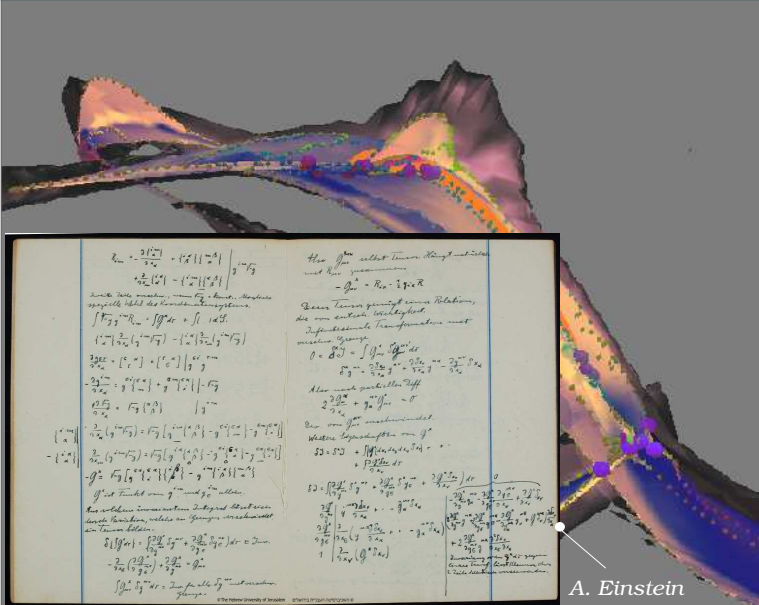
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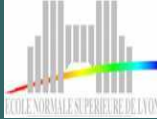


Éléments de
Physique Théorique et de Géométrie



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A. Einstein



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